Solutions to Homework 7

(1) a) $E = \dfrac{511\ \text{KeV}}{\sqrt{1-.18^2}} \approx 519.5\ \text{KeV}$

b) $E = \dfrac{511\ \text{KeV}}{\sqrt{1-.19^2}} \approx 520.5\ \text{KeV}$

c) $E = \dfrac{511\ \text{KeV}}{\sqrt{1-.98^2}} \approx 2567.9\ \text{KeV}$

d) $E = \dfrac{511\ \text{KeV}}{\sqrt{1-.99^2}} \approx 3622.4\ \text{KeV}$

(2) $p = m_0 c$

a) $m_0 c = \dfrac{m_0 u}{\sqrt{1-u^2/c^2}}$

$1 - \dfrac{u^2}{c^2} = \dfrac{u^2}{c^2} \Rightarrow$ $\boxed{\dfrac{u}{c} = \dfrac{1}{\sqrt{2}}}$

b) $E^2 = m_0^2 c^4 + p^2 c^2 = m_0^2 c^4 + (m_0 c)^2 c^2$

$E^2 = 2 m_0^2 c^4$

$\boxed{E = \sqrt{2}\, m_0 c^2}$

(2 cont.)

c) $KE = E - m_0c^2 = \boxed{(\sqrt{2} - 1)\, m_0c^2}$

(3) $E^2 = m_0^2c^4 + p^2c^2$

$(3m_0c^2)^2 = m_0^2c^4 + p^2c^2$

$p^2c^2 = 8m_0^2c^4$

$\boxed{p = \sqrt{8}\, m_0c}$

(4)

a) $6.9\,\mu m_0c = \dfrac{2.2\,\mu m_0c}{\sqrt{1 - v^2/c^2}}$

$\dfrac{1}{\sqrt{1 - v^2/c^2}} = \dfrac{6.9}{2.2} \approx 3.136$

$\dfrac{v}{c} \approx \sqrt{1 - \left(\dfrac{1}{3.136}\right)^2} \approx .948$

note that $v = u$, so

$\boxed{u \approx .948\, c}$

(4 cont)

b) $$E = \frac{mc^2}{\sqrt{1 - u^2/c^2}} = mc^2 \left(\frac{1}{\sqrt{1 - u^2/c^2}} \right)$$

$$\| \quad 3.136$$

$$E \simeq (105.7\ \text{MeV})(3.136)$$

$$E \simeq \boxed{331.5\ \text{MeV}}$$

c) $$E^2 = m_0^2 c^4 + p^2 c^2$$

$$pc = \sqrt{E^2 - m_0^2 c^4}$$

$$pc = \sqrt{331.5^2 - 105.7^2} \simeq 314.2\ \text{MeV}$$

$$\boxed{p \simeq 314.2\ \text{MeV}/c}$$

(5)

Before

after

(5 cont) Since the total momentum of the system equals zero, we have

a)

$$\frac{m_0 c/\sqrt{3}}{\sqrt{1-(\frac{1}{\sqrt{3}})^2}} = \frac{2 m_0 u}{\sqrt{1-u^2/c^2}}$$

$$\frac{c}{\sqrt{2}} = \frac{2u}{\sqrt{1-u^2/c^2}}$$

$$1 - \frac{u^2}{c^2} = \frac{8u^2}{c^2}$$

$$\boxed{u = \frac{c}{3}}$$

b) Since Energy is conserved

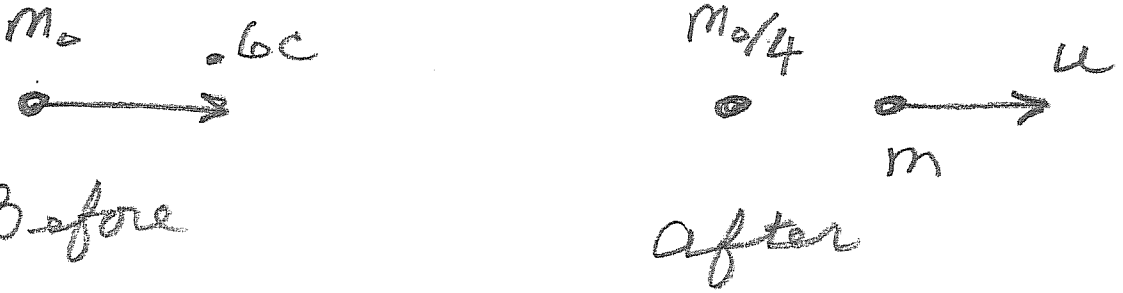
$$Mc^2 = E_1 + E_2$$

$$Mc^2 = \frac{m_0 c^2}{\sqrt{1-(\frac{1}{\sqrt{3}})^2}} + \frac{2 m_0 c^2}{\sqrt{1-(\frac{1}{3})^2}}$$

$$M = m_0\left(\sqrt{\frac{3}{2}} + \frac{3}{\sqrt{2}}\right)$$

$$\boxed{M = m_0\left(\frac{\sqrt{3}+3}{\sqrt{2}}\right)} \approx 3.34\, m_0$$

(6)



Before

after

Momentum conservation:

$$\frac{m_0(.6c)}{\sqrt{1-.6^2}} = \frac{mu}{\sqrt{1-u^2/c^2}}$$

$$\frac{3}{4}m_0 = \frac{m(u/c)}{\sqrt{1-u^2/c^2}}$$

Energy conservation:

$$\frac{m_0c^2}{\sqrt{1-.6^2}} = \frac{m_0}{4}c^2 + \frac{mc^2}{\sqrt{1-u^2/c^2}}$$

$$m_0\left(\frac{5}{4}-\frac{1}{4}\right) = \frac{m}{\sqrt{1-u^2/c^2}}$$

take the ratios of these 2 equations

$$3m_0 = \frac{m}{\sqrt{1-u^2/c^2}}$$

$$\frac{3}{4} = \frac{u}{c} \Rightarrow \boxed{u = \frac{3}{4}c}$$

(6 cont.)

(b) $$m_0 = \frac{m}{\sqrt{1 - u^2/c^2}}$$

$$m = \sqrt{1 - u^2/c^2}\, m_0$$

$$m = \sqrt{1 - \frac{9}{16}}\, m_0 = \boxed{\frac{\sqrt{7}}{4} m_0} \approx .66\, m_0$$

(7)

$p = 2 m_e c$ → m_e

Before

E_2 ← $P_2 = E_2/c$

E_1 → $P_1 = E_1/c$

Momentum conservation:

$$2 m_e c = \frac{E_1}{c} - \frac{E_2}{c} \Rightarrow \boxed{2 m_e c^2 = E_1 - E_2}$$

Energy conservation:

$$\sqrt{m_e^2 c^4 + (2 m_e c)^2 c^2} + m_e c^2 = E_1 + E_2$$

$$\sqrt{5}\, m_e c^2 + m_e c^2 = E_1 + E_2$$

$$\boxed{(1 + \sqrt{5})\, m_e c^2 = E_1 + E_2}$$

7 cont

Add the 2 equations:

$$(3+\sqrt{5})\, m_e c^2 = 2E_1$$

$$\boxed{E_1 = \left(\frac{3+\sqrt{5}}{2}\right) m_e c^2} \approx 2.62\, m_e c^2$$

$$E_2 = E_1 - 2 m_e c^2$$

$$\boxed{E_2 = \left(\frac{\sqrt{5}-1}{2}\right) m_e c^2} \approx .62\, m_e c^2$$