Physics 499 Winter 2016

Homework Assignment 2

Nuclear Shell Model

Due February 2nd

Problem : Nuclear Shell Model
Reference: Am. J. Phys. 68, 848 (Sept. 2000).

For this assignment you will determine (numerically) the allowed bound state energies for a neutron and a proton confined within a nucleus. To determine the allowed energies, solve the radial part of the discrete Schroedinger equation as derived in lecture:

$$u(i+1) = 2u(i) - u(i-1) + \Delta^2 \frac{l(l+1)}{r^2} u(i) + \frac{2m\Delta^2}{\hbar^2}(V(i) - E)u(i) \qquad (1)$$

for the energies E of all bound states.

For the strong potential that a neutron and a proton will experience, we will use a spherical square well potential:

$$\begin{aligned} V(r) &= -V_0 \qquad r \leq R \\ &= 0 \qquad r > R \end{aligned}$$

The proton will have in addition to the strong potential, an electrostatic potential. We will take this potential to be that due to a uniformly charged sphere of radius R and total charge Ze:

$$\begin{aligned} V_{Coulomb}(r) &= Ze^2 \frac{3R^2 - r^2}{2R^3} \qquad if\ r \leq R \\ &= \frac{Ze^2}{r} \qquad if\ r > R \end{aligned}$$

In your calculation, use the following values: Take the nuclear radius to be $R = 1.28A^{1/3}$ fm, $m_{neutron} \approx m_{proton} \approx 940\ MeV/c^2$; $\hbar c = 197.33\ MeV - fm$, and $V_0 = 50\ MeV$.

Using the method discussed in lecture, find all the allowed energy levels for neutrons and protons for the following values of A and l:

Neutrons

A	$l=0$	$l=1$	$l=2$	$l=3$
12				
16				
40				

Protons

A	$l=0$	$l=1$	$l=2$	$l=3$
12				
16				
40				

Note: There may be more than one energy for a particular value of l, and for the smaller nuclei there may be no bound states for larger l.

Your computer code should ask the user to input A, Z, l, V_0, and the starting value for the energy. Your code should output the bound state energy that is just above the starting energy. For the largest nucleus ($A = 40$), what is the ordering of the energy levels?

You should turn in (e-mail) two files: your computer code that will run in either gcc or ROOT, and a file discussing your results. For the discussion file, you can use straight text (*.txt) or latex. No *.doc files. Be sure your name is somewhere in each file you e-mail to me.