Visualizing the Thin-Lens Formula

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A common laboratory experiment in introductory physics classes is to have students form real images using thin lenses. An illuminated slit is often used as the object, and a thin lens focuses the light on a screen. The screen is adjusted until a sharp image is formed. The students measure the object distance o and the image distance i, which are the distances from the object and image to the lens. From these two measurements, the thin-lens equation[1]

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \qquad (1)$$

can be used to determine the focal length f of the lens. A well-known laboratory exercise is to have students take numerous measurements of the object distance and the corresponding image distance. For each of these object-image pairs, the focal length is calculated using Eq. (1). The focal lengths for each of the o and i combinations should be the same, and by comparing the calculated values of f, the student can test the validity of Eq. (1).

Although the above method is a useful exercise, we find that the thin-lens equation is best examined graphically. Equation (1) can be rewritten as:

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o}. \qquad (2)$$

For a particular lens, one can graph $1/o$, on the horizontal axis, versus $1/i$, on the vertical axis. A straight-line plot with a slope of -1 verifies Eq. (2) [and Eq. (1)]. The intercepts, horizontal or vertical axis, are equal to $1/f$. Graphing the data gives the students a visual verification of the thin-lens equation, as well as practice in graphical analysis.

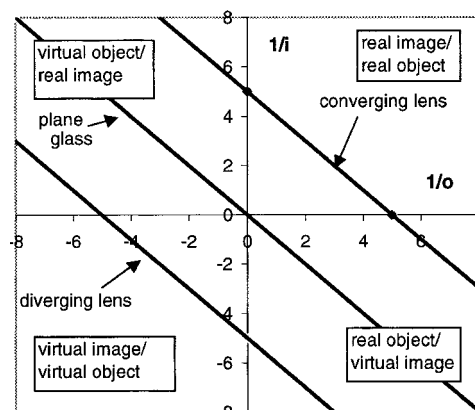The $1/o – 1/i$ plane is a convenient canvas for instruction in the basics of thin lenses. Points in

Fig. 1. The 1/o – 1/i plane. The lines drawn would represent a particular converging lens, diverging lens, or plane glass plate.
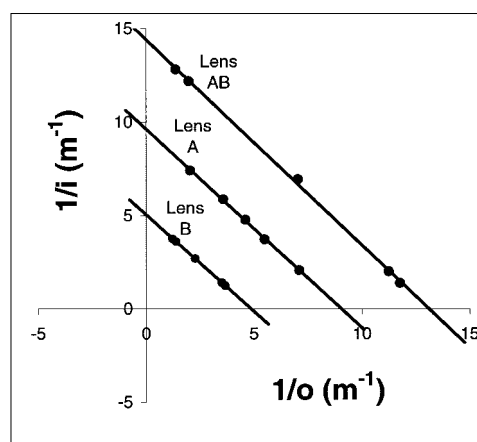

Fig. 2. Plot of image and object data taken in a student laboratory class. Two simple lenses, lens A and lens B, are plotted along with the combination lens AB.

the first quadrant correspond to real objects and real images. Real objects and virtual images are plotted in the fourth quadrant. Virtual objects and virtual images lie in the third quadrant, and virtual objects and real images are located in the second quadrant.[2] (See Fig. 1.) All lenses, converging or diverging, are represented by lines

that have a slope of negative one. The intercepts of the lines are equal to the reciprocal of the focal length. Lines for converging lenses will intersect the positive horizontal (and vertical) axes, and diverging lenses will intersect the negative horizontal (and vertical) axes as shown in Fig. 1. The line representing a plane glass plate, $f \to \infty$, will intersect the origin.

In our experiment, we have the students take image and object distance data for two different converging lenses, named lens A and lens B. The students then place the two lenses together and take image and object data for the combination, which we call lens AB. Data taken in our student laboratory for lens A, lens B, and the combination AB are plotted in Fig. 2. The straight-line plots, with an approximate slope of –1, for lenses A and B indicate that they follow Eq. (1). The straight-line plot, with an approximate slope of –1, for the combination AB shows that two thin lenses in contact act as a single thin lens. The student can also check that the focal length of the combination, f_{AB}, is related to the focal length of lens A, f_A, and lens B, f_B, by the relation:

$$\frac{1}{f_{AB}} = \frac{1}{f_A} + \frac{1}{f_B}. \tag{3}$$

Equation (3) has a simple interpretation in Fig. 2: the horizontal intercept of the combination AB is equal to the sum of the horizontal intercepts of lenses A and B.

The data our students obtain for the experiment are generally of good quality. In Fig. 2, least-squares fits to the data give slopes of –0.99, –1.06, and –1.10 for lenses A, B, and the combination AB. The average of the intercepts are approximately $1/f_A = 5$ m^{-1}, $1/f_B = 9$ m^{-1}, and $1/f_{AB} = 14$ m^{-1}. The students are usually impressed with their straight-line results and the verification of Eq. (3) using the intercepts. Also, they can usually discover the symmetry in the object-image distances: that is, if (x, y) is a point for a particular lens, then (y, x) is also.

The inverse focal length $1/f$ is referred to as the refracting power P of the lens. The power of a lens is a common term for optometrists, but is usually not used in introductory physics texts. If

f has units of meters, the power $1/f$ has units of diopters D. In terms of the refracting power of two thin lenses in contact, Eq. (3) states that the refracting power of two thin lenses in contact is equal to the sum of the refracting powers of the individual lenses. The inverse image and object distances are called the vergence, V[3,4]. If V represents the vergence of the object and V' the vergence of the image, Eq. (1) becomes $V + V' = P$.

We note that a straight-line graph of $1/o$ versus $1/i$ does not directly verify any fundamental law of physics. The relevant physics at work is Snell's law of refraction, which derives from either the properties of waves propagating across boundaries, Fermat's principle, or Maxwell's equations. The simplicity of Eq. (1) is due to the shape of the lens. If the thickness of the lens is small compared to its diameter and the surfaces are near spherical, then the image will be focused and Eq. (1) is a good approximation.

We find that our students enjoy the experiment more when the image and object data are graphed. The data are generally of good enough quality to give a clear visual verification of both Eqs. (1) and (3) on the same graph. It is always worthwhile to use graphing and the associated data analysis techniques in our laboratory classes. The thin-lens experiment is another opportunity to apply these methods.

Acknowledgments

References

1. The thin-lens equation is found in nearly every introductory physics text. See for example D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics* (Wiley, New York, 1997).

2. An object is virtual (object behind the lens) when rays converge toward it. This can occur in a compound lens system when a lens intercepts the rays before a real object can be formed.

3. F. L. Pedrotti and L. S. Pedrotti, *Introduction to Optics* (Prentice-Hall, New Jersey, 1993).

4. Kenneth N. Ogle, *Optics: An Introduction for Ophthalmologists* (Charles C. Thomas, Bannerston House, Springfield, IL, 1961).